A Formal Web Services Architecture Model for Changing PUSH/PULL Data Transfer

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A Appendix: Equivalence of JAX-RS Prototype and Data Transfer Architecture Model

In this appendix, we show the detailed ones of the proofs shown in Sec. 6 in the conference paper.

A.1 Equivalence of PUSH-first JAX-RS Prototype

First, we prove the equivalence between a data transfer architecture model and the generated PUSH-first prototype. Let $\mathcal{R}$ be any data transfer architecture model, r be a resource in $\mathcal{R}$ and $\mathcal{P}_{\mathcal{R}}$ be a JAX-RS prototype generated from $\mathcal{R}$. Then, $\Phi(\mathcal{P}_{\mathcal{R}}, r)$ represents the class in $\mathcal{P}_{\mathcal{R}}$ corresponding to r, $\mathcal{P}_{\mathcal{R}} \setminus \Phi(\mathcal{P}_{\mathcal{R}}, r)$ represents the JAX-RS prototype obtained by removing $\Phi(\mathcal{P}_{\mathcal{R}}, r)$ from $\mathcal{P}_{\mathcal{R}}$, and $\mathrm{trim}_{\mathcal{R}}(\cdot)$ represents an operation on a JAX-RS prototype that removes all calls and all method bodies that are not relevant to $\mathcal{R}$. More specifically, for any resource r that is not contained in $\mathcal{R}$ and the class P_r corresponding to r, $\mathrm{trim}_{\mathcal{R}}(\cdot)$ removes all calls to the update method of P_r and all method bodies of update methods called only by P_r.

Lemma 1. *Given an arbitrary valid data transfer architecture model* $\mathcal{R} = \langle R, C, \rho, T, \tau, \mu, \Sigma, \Delta, \Gamma, s_0\rangle$, *for any state* s *and any resource* $r \in R$, $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s, x_1, \ldots, x_L) \overset{\mathrm{get}(r)/s(r)}{\Longrightarrow} \mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s, x_1, \ldots, x_L)$.

Proof. The lemma directly follows from the implementation of the getter method of $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}$. □

Lemma 2. *Given an arbitrary valid data transfer architecture model* $\mathcal{R} = \langle R, C, \rho, T, \tau, \mu, \Sigma, \Delta, \Gamma, s_0\rangle$, *let* $\mathcal{R}'$ *be the data transfer architecture model obtained by removing a resource* $\tilde{r}$ *that satisfies* $\mathrm{Out}_{\mathrm{C}}(\tilde{r}) = \emptyset$ *from* $\mathcal{R}$. *Let* $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'} = \mathrm{trim}_{\mathcal{R}'}(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}} \setminus \Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r}))$. *Then, for any state* s *of* $\mathcal{R}$ *and any input message* $\widehat{\langle m, c_{\mathrm{I/O}}\rangle}$ *for* $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ *and* $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s)$, *if there exists a state* s' *of* $\mathcal{R}$ *and*

$$\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s) \overset{\widehat{\langle m, c_{\mathrm{I/O}}\rangle}}{\Longrightarrow} \mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s'),$$

then

$$\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s) \overset{\widehat{\langle m, c_{\mathrm{I/O}}\rangle}}{\Longrightarrow} \mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s')$$

holds.

In addition, for any resource $r \in R \setminus \{\tilde{r}\}$*, the control reaches* $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ *from* $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ *by* $\widehat{\langle m, c_{\mathrm{I/O}} \rangle}$ *if and only if the control reaches* $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ *from* $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s)$ *by* $\widehat{\langle m, c_{\mathrm{I/O}} \rangle}$*.*

Proof. On the assumption that the condition part of the lemma holds, we prove the conclusion part of the lemma for two cases that the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r})$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ by $\widehat{\langle m, c_{\mathrm{I/O}} \rangle}$ and that the control does not reach $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r})$ on the same condition.

If the control does not reach $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r})$, then

$$(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}} \setminus \Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r}))(s) \overset{\widehat{\langle m, c_{\mathrm{I/O}} \rangle}}{\Longrightarrow} (\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}} \setminus \Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r}))(s')$$

holds, and since this relation is preserved by $\mathrm{trim}_{\mathcal{R}'}$, also

$$\begin{aligned}
&\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s) = \mathrm{trim}_{\mathcal{R}'}(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}} \setminus \Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r}))(s) \\
&\overset{\widehat{\langle m, c_{\mathrm{I/O}} \rangle}}{\Longrightarrow} \mathrm{trim}_{\mathcal{R}'}(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}} \setminus \Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r}))(s') = \mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s')
\end{aligned}$$

holds. By this and that resource r that satisfies $r \in R \setminus \{\tilde{r}\}$ is contained in $\mathcal{R}'$, the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ by $\widehat{\langle m, c_{\mathrm{I/O}} \rangle}$ if and only if it reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s)$ by $\widehat{\langle m, c_{\mathrm{I/O}} \rangle}$.

If the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r})$, then a method of $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r})$ is called just before the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r})$. From the definition of $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}$, the called method is the update method of $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r})$. Since every call to the update method is removed by applying $\mathrm{trim}_{\mathcal{R}'}$, any program execution from $\mathrm{trim}_{\mathcal{R}'}(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}} \setminus \Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r}))(s)$ never terminates at a call to the update method. On the other hand, since there is no $c \in C$ such that $\tilde{r} \in \rho(c, \mathrm{I})$, any call of an update method from $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r})$ is not contained in any program execution from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$. Thus, the program execution from $\mathrm{trim}_{\mathcal{R}'}(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}} \setminus \Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r}))(s)$ that is obtained by removing the class corresponding to $\tilde{r}$ is never shortened from the original transition sequence from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$. Therefore,

$$\begin{aligned}
&\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s) = \mathrm{trim}_{\mathcal{R}'}(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}} \setminus \Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r}))(s) \\
&\overset{\widehat{\langle m, c_{\mathrm{I/O}} \rangle}}{\Longrightarrow} \mathrm{trim}_{\mathcal{R}'}(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}} \setminus \Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r}))(s') = \mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s')
\end{aligned}$$

holds, and for any r that satisfies $r \in R \setminus \{\tilde{r}\}$, the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ by $\widehat{\langle m, c_{\mathrm{I/O}} \rangle}$ if and only if it reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s)$ by $\widehat{\langle m, c_{\mathrm{I/O}} \rangle}$. □

Lemma 3. *Given an arbitrary valid data transfer architecture model* $\mathcal{R} = \langle R, C, \rho, T, \tau, \mu, \Sigma, \Delta, \Gamma, s_0 \rangle$*, for any state* s*, input channel* $c_{\mathrm{I/O}}$ *and message* m*,* $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s) \overset{\widehat{\langle m, c_{\mathrm{I/O}} \rangle}}{\Longrightarrow} \mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s')$ *iff* $s \xrightarrow[\mathcal{R}]{\langle m, c_{\mathrm{I/O}} \rangle} s'$*.*

In addition, let π *be a message assignment when* $s \xrightarrow[\mathcal{R}]{\langle m, c_{\mathrm{I/O}} \rangle} s'$ *holds. Then, for any resource* $r \in R$*,*

1. *if there exists a channel $c \in C$ such that $r \in \rho(c, \mathrm{O})$ and $\pi(c) \neq e_c$, then the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ by $\langle \widehat{m, c_{\mathrm{I/O}}} \rangle$,*
2. *otherwise the control does not reach $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ by $\langle \widehat{m, c_{\mathrm{I/O}}} \rangle$ and $s(r) = s'(r)$ holds.*

Proof. The lemma is proved by induction on the number $n = |R|$ of the resources.
(**basis**) We assume that $\mathcal{R}$ has one resourse and only input channels, that is, $R = \{r\}$, $C = C_{\mathrm{I/O}}$ and $\rho(c, \mathrm{O}) = \{r\}$ for any $c \in C$. Let $C = \{c_1, \ldots, c_n\}$. Here, with respect to $\mathcal{R}$, if $s \xrightarrow[\mathcal{R}]{\langle m, c_i \rangle} s'$ holds (for some i s.t. $1 \leq i \leq n$), then $s'(r) = \delta^{c_i, \mathrm{O}}_r(s(r), m)$ holds by equation (2). On the other hand, with respect to the PUSH-first JAX-RS prototype $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ of $\mathcal{R}$,

$$\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s) \xrightarrow{\mathrm{input\_on\_}c_i(r,m)/\mathrm{void}} \mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s'')$$

holds by the generation of the PUSH-first prototype, where $s''(r) = \delta^{c_i, \mathrm{O}}_r(s(r), m)$. Here, since $\mathcal{R}$ has only one resource r, $s' = s''$ follows from $s'(r) = s''(r)$, and the first part of the lemma holds. Furthermore, the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ because an input method of $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ is called by input_on_$c_i(r, m)$/void. Since $r \in \rho(c_i, \mathrm{O})$ and $\pi(c_i) = m \neq e_{c_i}$, also the second part of the lemma holds.
(**induction step**) For $\mathcal{R}$, there exists $\mathcal{R}' = \langle R', C', \rho', T, \tau, \mu, \Sigma, \Delta', \Gamma, s'_0 \rangle$ that satisfies the following conditions (see Fig. 1) and the induction hypothesis holds for $\mathcal{R}'$.

- There exists an appropriate $r \in R$, there is no $c' \in C$ such that $r \in \rho(c', \mathrm{I})$ and $R' = R \setminus \{r\}$ holds.
- For any $c \in C$ that satisfies $r \in \rho(c, \mathrm{O})$,
  - if $\rho(c, \mathrm{O}) = \{r\}$ holds, then $c \notin C'$ holds, and
  - if $\rho(c, \mathrm{O}) \supset \{r\}$ holds, then $c \in C'$ and $\rho'(c, \mathrm{O}) = \rho(c, \mathrm{O}) \setminus \{r\}$ hold.

First, we prove the lemma with respect to the reachability of the control. We prove this for two cases that there exists $c \in C$ such that $r \in \rho(c, \mathrm{O})$ and $\pi(c) \neq e_c$, and that there does not exist.
(i) If there exists some $c \in C$ such that $r \in \rho(c, \mathrm{O})$ and $\pi(c) \neq e_c$:
Also, we prove the case for two cases that $c \in C_{\mathrm{I/O}}$ and that $c \notin C_{\mathrm{I/O}}$.
(i.a) If $c \in C_{\mathrm{I/O}}$:
Since $c \in C_{\mathrm{I/O}}$ and $\pi(c) \neq e_c$, $c = c_{\mathrm{I/O}}$ and $\pi(c) = m$. By the generation of the PUSH-first prototype, there exists an input method `input_on_`$c_{\mathrm{I/O}}$ in $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$. Thus, the method is called by input_on_$c_{\mathrm{I/O}}(r, m)$/void, and the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ by $\langle \widehat{m, c_{\mathrm{I/O}}} \rangle$.
(i.b) If $c \notin C_{\mathrm{I/O}}$:
There exist channel $c' \in C$ and resource r_j in $\mathcal{R}$ that satisfy $r_j \in \rho(c, \mathrm{I})$, $r_j \in \rho(c', \mathrm{O})$ and $\pi(c') \neq e_{c'}$ from $\pi(c) \neq e_c$ and the definition of π. By the induction hypothesis, the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}, r_j)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s)$ by $\langle \widehat{m, c_{\mathrm{I/O}}} \rangle$. By lemma 2, also, the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r_j)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ by $\langle \widehat{m, c_{\mathrm{I/O}}} \rangle$.

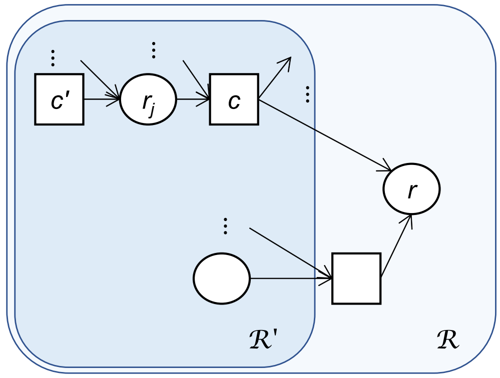

Fig. 1: Proof of lemma 3

On the other hand, by the generation of the PUSH-first prototype, $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r_j)$ has an update method `update_from_`r' where $r' \in \rho(c', \mathrm{I})$ if $c' \notin C_{\mathrm{I/O}}$, or an input method `input_on_`$c'$ if $c' \in C_{\mathrm{I/O}}$. In addition, both of these methods call an update method `update_from_`r_j in $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$. Since the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r_j)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ by $\widehat{\langle m, c_{\mathrm{I/O}}\rangle}$, `update_from_`$r_j$ in $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ is called and the control also reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$.
(ii) If there does not exist $c \in C$ such that $r \in \rho(c, \mathrm{O})$ and $\pi(c) \neq e_c$:
For any c such that $r \in \rho(c, \mathrm{O})$, $\pi(c) = e_c$ holds, and $G_{\mathcal{R}, c_{\mathrm{I/O}}}$ does not contain c by the definition of π. It is clear that the control does not reach $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ since $c \neq c_{\mathrm{I/O}}$ even if $c \in C_{\mathrm{I/O}}$. In addition, $G_{\mathcal{R}, c_{\mathrm{I/O}}}$ does not contain any r_j and any c' that satisfy $r_j \in \rho(c, \mathrm{I})$ and $r_j \in \rho(c', \mathrm{O})$. Hence, $\pi(c') = e_{c'}$ holds for any c', and by the induction hypothesis, there does not exist $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}, r_j)$ that the control reaches from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s)$. Furthermore, by lemma 2, also, there does not exist $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r_j)$ that the control reaches from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$. Therefore, the control does not reach $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ by $\widehat{\langle m, c_{\mathrm{I/O}}\rangle}$.

Second, we prove the lemma with respect to equivalence of the transition relations. We prove this for two cases that there exists $c \in C$ such that $r \in \rho(c, \mathrm{O})$ and $\pi(c) \neq e_c$ and that there does not exist.
(i) If there exists $c \in C$ such that $r \in \rho(c, \mathrm{O})$ and $\pi(c) \neq e_c$:
Let $s_{\mathcal{R}} = s$ and $s'_{\mathcal{R}} = s'$ to distinguish the states of $\mathcal{R}$ from those of $\mathcal{R}'$.
(i.a) If $c \notin C_{\mathrm{I/O}}$:

$$\delta^{c,O}_{r}(s_{\mathcal{R}}(r), \pi(c)) = s'_{\mathcal{R}}(r) \tag{6}$$

holds by equation (2) because $G_{\mathcal{R}, c_{\mathrm{I/O}}}$ contains c. Also, there exist r_j and c' that satisfy $r_j \in \rho(c, \mathrm{I})$, $r_j \in \rho(c', \mathrm{O})$ and $\pi(c') \neq e_{c'}$, and

$$\delta^{c,I}_{r_j}(s_{\mathcal{R}}(r_j), \pi(c)) = s'_{\mathcal{R}}(r_j) \tag{7}$$

holds. Let y' be the next state of r_j in $\mathcal{R}'$. Then,

$$y' = \delta^{c',O}_{r_j}(s_{\mathcal{R}'}(r_j), \pi(c')) = s'_{\mathcal{R}'}(r_j) = s'_{\mathcal{R}}(r_j) \tag{8}$$

holds since $s_{\mathcal{R}'}(r_j) = s_{\mathcal{R}}(r_j)$. On the other hand, with respect to $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}$, the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}, r_j)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s_{\mathcal{R}'})$ by the induction hypothesis since $\pi(c') \neq e_{c'}$. Also with respect to $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}$, the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r_j)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s_{\mathcal{R}})$ by lemma 2. Thus, the update method of $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ is called by $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r_j)$. Let x' be the state just after the update method is called. Then,

$$x' = \delta^{c,\mathrm{O}}_{r}(s_{\mathcal{R}}(r), m_c) \tag{9}$$

holds for some message m_c by the generation of the PUSH-first prototype. Furthermore,

$$y' = \delta^{c,I}_{r_j}(s_{\mathcal{R}}(r_j), m_c) \tag{10}$$

holds by the induction hypothesis of equivalence of the transition relations. By equations (8) and (10), we have

$$s'_{\mathcal{R}}(r_j) = \delta^{c,I}_{r_j}(s_{\mathcal{R}}(r_j), m_c). \tag{11}$$

In addition, by comparing equations (7) and (11), and equation (1),

$$m_c = \pi(c)$$

holds. By equation (6) we have,

$$s'_{\mathcal{R}}(r) = \delta^{c,O}_{r}(s_{\mathcal{R}}(r), m_c). \tag{12}$$

Since by equations (9) and (12),

$$x' = s'_{\mathcal{R}}(r) \tag{13}$$

holds, $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s_{\mathcal{R}}) \overset{\widehat{\langle m, c_{\mathrm{I/O}} \rangle}}{\Longrightarrow} \mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s'_{\mathcal{R}})$ holds.
(i.b) If $c \in C_{\mathrm{I/O}}$:

$$\delta^{c,O}_{r}(s_{\mathcal{R}}(r), m) = s'_{\mathcal{R}}(r) \tag{14}$$

holds by equation (2) since $G_{\mathcal{R},c_{\mathrm{I/O}}}$ contains c by $c = c_{\mathrm{I/O}}$. Then,

$$x' = \delta^{c,\mathrm{O}}_{r}(s_{\mathcal{R}}(r), m) \tag{15}$$

holds by the generation of the PUSH-first prototype because the input method of $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ is called from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s_{\mathcal{R}})$ (where x' is the state of r after the method is called). By equations (14) and (15), we have $x' = s'_{\mathcal{R}}(r)$.
(ii) If there does not exist $c \in C$ such that $r \in \rho(c, \mathrm{O})$ and $\pi(c) \neq e_c$:
$s(r) = s'(r)$ holds for $\mathcal{R}$ by the definition of π.
(ii.a) If $c \in C_{\mathrm{I/O}}$:
The control does not reach $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ because $G_{\mathcal{R},c_{\mathrm{I/O}}}$ does not contain c and $c \neq c_{\mathrm{I/O}}$.
(ii.b) If $c \notin C_{\mathrm{I/O}}$:
$G_{\mathcal{R},c_{\mathrm{I/O}}}$ does not contain any resource r_j and any channel c' such that $r_j \in \rho(c, \mathrm{I})$ and $r_j \in \rho(c', \mathrm{O})$. Hence, the control does not reach any $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}, r_j)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}'}(s)$ by the induction hypothesis and $\pi(c') = e'_c$. Also, the control does not

reach any $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r_j)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ by lemma 2. Therefore, the control does not reach $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, r)$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ and

$$s(r) = s'(r) = s''(r) \tag{16}$$

holds for the transition of $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}$ from state s to s''.
From (i) and (ii), for any state s, input channel $c_{\mathrm{I/O}}$ and message m,
$\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s) \stackrel{\widehat{\langle m, c_{\mathrm{I/O}}\rangle}}{\Longrightarrow} \mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s')$ iff $s \xrightarrow[\mathcal{R}]{\langle m, c_{\mathrm{I/O}}\rangle} s'$. □

Theorem 1. *Let $\mathcal{R} = \langle R, C, \rho, D, \tau, \mu, \Delta, s_0\rangle$ be an arbitrary valid data transfer architecture model. Then, for any input sequence σ, $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s_0) \stackrel{\widehat{\sigma}}{\Longrightarrow} \mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ iff $s_0 \underset{\mathcal{R}}{\stackrel{\sigma}{\Longrightarrow}} s$.*

Proof. The lemma follows from lemmas 1 and 3. □

A.2 Equivalence of Arbitrary JAX-RS Prototype

Next, we prove the equivalence between the generated PUSH-first prototype and any PULL-containing prototype.

Lemma 4. *Given an arbitrary valid data transfer architecture model $\mathcal{R} = \langle R, C, \rho, T, \tau, \mu, \Sigma, \Delta, \Gamma, s_0\rangle$, let $\mathcal{P}_{\mathcal{R}}$ be an arbitrary JAX-RS prototype that is generated from $\mathcal{R}$ and satisfies conditions 1 and 2. Also, let $\mathcal{R}'$ be the data transfer architecture model obtained by removing a resource $\tilde{r}$ that satisfies $\mathrm{Out}_{\mathrm{C}}(\tilde{r}) = \emptyset$ from $\mathcal{R}$. Let $\mathcal{P}_{\mathcal{R}'} = \mathrm{trim}_{\mathcal{R}'}(\mathcal{P}_{\mathcal{R}} \setminus \Phi(\mathcal{P}_{\mathcal{R}}, \tilde{r}))$. Then, for any state s of $\mathcal{R}$ and any input message sequence σ for $\mathcal{P}_{\mathcal{R}}$ and $\mathcal{P}_{\mathcal{R}'}$, if there exists some state s' of $\mathcal{R}$ and $\mathcal{P}_{\mathcal{R}}(s) \stackrel{\sigma}{\Longrightarrow} \mathcal{P}_{\mathcal{R}}(s')$ holds, then $\mathcal{P}_{\mathcal{R}'}(s) \stackrel{\sigma}{\Longrightarrow} \mathcal{P}_{\mathcal{R}'}(s')$.*

Proof. The lemma is proved by induction on the length n of σ.
(**basis**)
If $n = 0$, then the lemma follows from the definition of $\mathcal{P}_{\mathcal{R}'}$.
(**induction step**)
If $n > 0$, then first, let $\sigma = \sigma' \langle m, c_{\mathrm{I/O}}\rangle$. Next, similar to lemma 2, we prove the induction step for two cases that the control reaches $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r})$ from $\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}(s)$ by $\langle m, c_{\mathrm{I/O}}\rangle$ and that the control does not reach $\Phi(\mathcal{P}^{\mathrm{PSH}}_{\mathcal{R}}, \tilde{r})$ on the same condition. □

Lemma 5. *Let $\mathcal{R} = \langle R, C, \rho, T, \tau, \mu, \Sigma, \Delta, \Gamma, s_0\rangle$ be an arbitrary valid data transfer architecture model and $\mathcal{R}'$ be the data transfer model obtained by removing $\tilde{r}$ that satisfies $\mathrm{Out}_{\mathrm{C}}(\tilde{r}) = \emptyset$ from $\mathcal{R}$. Also, let $R' = R \setminus \{\tilde{r}\}$ be the set of resource of $\mathcal{R}'$. Then, any JAX-RS prototypes $\mathcal{P}_{\mathcal{R}}$ and $\mathcal{P}'_{\mathcal{R}}$ that are generated from $\mathcal{R}$ and satisfy conditions 1 and 2 satisfy*

$$\begin{aligned}\forall_{\sigma}.\forall_{r\in R}.\ &\{s_r \mid \exists_s.\mathcal{P}_{\mathcal{R}}(s_0) \stackrel{\sigma}{\Rightarrow} \mathcal{P}_{\mathcal{R}}(s) \stackrel{\mathrm{get}(r)/s_r}{\Longrightarrow} \mathcal{P}_{\mathcal{R}}(s)\} \\ &= \{s'_r \mid \exists_{s'}.\mathcal{P}'_{\mathcal{R}}(s_0) \stackrel{\sigma}{\Rightarrow} \mathcal{P}'_{\mathcal{R}}(s') \stackrel{\mathrm{get}(r)/s'_r}{\Longrightarrow} \mathcal{P}'_{\mathcal{R}}(s')\}\end{aligned}$$

if any JAX-RS prototypes $\mathcal{P}_{\mathcal{R}'}$ and $\mathcal{P}'_{\mathcal{R}'}$ that are generated from $\mathcal{R}'$ and satisfy conditions 1 and 2 satisfy

$$\forall_\sigma.\forall_{r\in R'}.\ \{s_r \mid \exists_s.\mathcal{P}_{\mathcal{R}'}(s_0) \overset{\sigma}{\Rightarrow} \mathcal{P}_{\mathcal{R}'}(s) \overset{\mathrm{get}(r)/s_r}{\Longrightarrow} \mathcal{P}_{\mathcal{R}'}(s)\}$$
$$= \{s'_r \mid \exists_{s'}.\mathcal{P}'_{\mathcal{R}'}(s_0) \overset{\sigma}{\Rightarrow} \mathcal{P}'_{\mathcal{R}'}(s') \overset{\mathrm{get}(r)/s'_r}{\Longrightarrow} \mathcal{P}'_{\mathcal{R}'}(s')\}.$$

Proof. On the assumption that the condition part of the lemma holds, we prove its conclusion part by induction on the length n of σ in the conclusion part.
(**basis**)
If $n = 0$, then since lemma 4 holds, it is sufficient to show that $s_{\tilde{r}}^{\mathcal{P},\epsilon} = s_{\tilde{r}}^{\mathcal{P}',\epsilon}$ holds for $\tilde{r}$ if $\mathcal{P}_{\mathcal{R}}(s_0) \overset{\mathrm{get}(\tilde{r})/s_{\tilde{r}}^{\mathcal{P},\epsilon}}{\Longrightarrow} \mathcal{P}_{\mathcal{R}}(s_0)$ and $\mathcal{P}'_{\mathcal{R}}(s_0) \overset{\mathrm{get}(\tilde{r})/s_{\tilde{r}}^{\mathcal{P}',\epsilon}}{\Longrightarrow} \mathcal{P}'_{\mathcal{R}}(s_0)$ hold.

First, consider a channel $c \in C$ such that $\tilde{r} \in \rho(c, \mathrm{I})$, and let $\{r_1, \ldots, r_l\} = \rho(c, \mathrm{I})$. Then, in $\mathcal{P}_{\mathcal{R}}$, if $\langle r_j, c\rangle \notin E^{\mathrm{PLL}}$ for any r_j $(1 \le j \le l)$, then by the generation of the PUSH-first prototype,

$$s_{\tilde{r}}^{\mathcal{P},\epsilon} = s_0(\tilde{r}) \tag{17}$$

holds. Next, in $\mathcal{P}'_{\mathcal{R}}$, if $\langle r_j, c\rangle \in E^{\mathrm{PLL}}$ for some r_j $(1 \le j \le l)$, then by the generation of a PULL-containing prototype,

$$s_{\tilde{r}}^{\mathcal{P}',\epsilon} = f_{\tilde{r}}(s_0(r_1), \ldots, s_0(r_l)). \tag{18}$$

Therefore, by equations (17), (18) and equation (3) in condition 2, $s_{\tilde{r}}^{\mathcal{P},\epsilon} = s_{\tilde{r}}^{\mathcal{P}',\epsilon}$ holds.

On the other hand, also in $\mathcal{P}_{\mathcal{R}}$, if $\langle r_i, c\rangle \in E^{\mathrm{PLL}}$ for some r_i $(1 \le i \le l)$, then by the generation of a PULL-containing prototype and the initial conditions of the caches within $\tilde{r}$, $s_{\tilde{r}}^{\mathcal{P},\epsilon} = f_{\tilde{r}}(s_0(r_1), \ldots, s_0(r_l)) = s_{\tilde{r}}^{\mathcal{P}',\epsilon}$ holds.
(**induction step**)
If $n > 0$, then let $\sigma = \sigma'\ \langle m, c_{\mathrm{I/O}}\rangle$. Note that the length of σ' is $n - 1$. Since lemma 4 holds, it is sufficient to show that only for $\tilde{r}$, the conclusion part of the lemma holds. In the following, we show that $s_{\tilde{r}}^{\mathcal{P},\sigma} = s_{\tilde{r}}^{\mathcal{P}',\sigma}$ where $s_{\tilde{r}}^{\mathcal{P},\sigma}$ and $s_{\tilde{r}}^{\mathcal{P}',\sigma}$ are the responses of requesting $\mathrm{get}(\tilde{r})$ just after inputting σ to JAX-RS prototypes $\mathcal{P}_{\mathcal{R}}$ and $\mathcal{P}'_{\mathcal{R}}$, respectively.

Consider the case that there exists exactly one $c \in C$ such that $\tilde{r} \in \rho(c, \mathrm{O})$ and let $\{r_1, \ldots, r_l\} = \rho(c, \mathrm{I})$ (see Fig. 2). We prove the above equation for the following three cases.

1. With respect to both $\mathcal{P}_{\mathcal{R}}$ and $\mathcal{P}'_{\mathcal{R}}$, $\langle r_j, c\rangle \notin E^{\mathrm{PLL}}$ for every r_j $(1 \le j \le l)$.
2. With respect to only $\mathcal{P}_{\mathcal{R}}$, $\langle r_j, c\rangle \notin E^{\mathrm{PLL}}$ for every r_j $(1 \le j \le l)$.
3. With respect to both $\mathcal{P}_{\mathcal{R}}$ and $\mathcal{P}'_{\mathcal{R}}$, $\langle r_j, c\rangle \in E^{\mathrm{PLL}}$ for some r_j $(1 \le j \le l)$.

Note that if there exists more than one $c \in C$ such that $\tilde{r} \in \rho(c, \mathrm{O})$, then by condition 1-(2), for any c, $\langle r_j, c\rangle \notin E^{\mathrm{PLL}}$ must be satisfied for any r_j $(1 \le j \le l)$, and thus, this condition corresponds to the above 1).

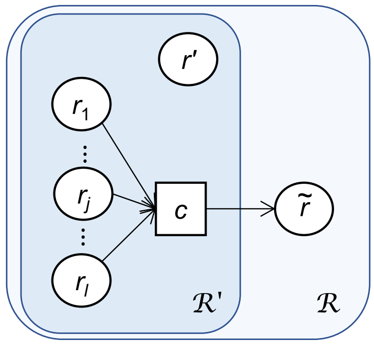

Fig. 2: Proof of lemma 5

First, we consider the most typical case 2). In this case, the class $\Phi(\mathcal{P}_\mathcal{R}, \tilde{r})$ of $\mathcal{P}_\mathcal{R}$ has a state field. Thus, by the generation of PUSH-first prototype, the latest state of $\Phi(\mathcal{P}_\mathcal{R}, \tilde{r})$ just after inputting σ' becomes $s_{\tilde{r}}^{\mathcal{P},\sigma'}$, and by the induction hypothesis, we have

$$s_{\tilde{r}}^{\mathcal{P},\sigma'} = s_{\tilde{r}}^{\mathcal{P}',\sigma'}. \tag{19}$$

On the other hand, with respect to $\mathcal{P}'_\mathcal{R}$, since $\langle r_j, c\rangle \in E^{\mathrm{PLL}}$ for some r_j $(1 \leq j \leq l)$, by the generation of PUSH-first prototype, the latest state of $\Phi(\mathcal{P}'_\mathcal{R}, r_j)$ just after inputting σ' to $\mathcal{P}'_\mathcal{R}$ becomes $s_{r_j}^{\mathcal{P}',\sigma'}$. As $r_j \in R'$, the condition part of the lemma holds for σ' and r_j, and by lemma 4, we have

$$s_{r_j}^{\mathcal{P}',\sigma'} = s_{r_j}^{\mathcal{P},\sigma'}. \tag{20}$$

Furthermore, by the generation of a PULL-containing prototype,

$$f_{\tilde{r}}(s_{r_1}^{\mathcal{P}',\sigma'}, \ldots, s_{r_l}^{\mathcal{P}',\sigma'}) = s_{\tilde{r}}^{\mathcal{P}',\sigma'} \tag{21}$$

holds. Next, consider the state of $\mathcal{P}_\mathcal{R}$ just after σ is input. Then, by the generation of PUSH-first prototype, the response $s_{\tilde{r}}^{\mathcal{P},\sigma}$ of the getter method of $\Phi(\mathcal{P}_\mathcal{R}, \tilde{r})$ becomes

$$s_{\tilde{r}}^{\mathcal{P},\sigma} = \delta_{\tilde{r}}^{c,\mathrm{O}}(s_{\tilde{r}}^{\mathcal{P},\sigma'}, m) \tag{22}$$

and the response of the getter method of $\Phi(\mathcal{P}_\mathcal{R}, r_j)$ becomes

$$s_{r_j}^{\mathcal{P},\sigma} = \delta_{r_j}^{c,\mathrm{I}}(s_{r_j}^{\mathcal{P},\sigma'}, m) \tag{23}$$

for an appropriate message m on channel c. Moreover, consider the state of $\mathcal{P}'_\mathcal{R}$ just after σ is input. Then, by the generation of a PULL-containing prototype, the response $s_{\tilde{r}}^{\mathcal{P}',\sigma}$ of the getter method of $\Phi(\mathcal{P}'_\mathcal{R}, \tilde{r})$ becomes

$$s_{\tilde{r}}^{\mathcal{P}',\sigma} = f_{\tilde{r}}(s_{r_1}^{\mathcal{P}',\sigma}, \ldots, s_{r_l}^{\mathcal{P}',\sigma}). \tag{24}$$

As $r_j \in R'$, the condition part of the lemma holds for σ and r_j, and by lemma 4,

$$s_{r_j}^{\mathcal{P}',\sigma} = s_{r_j}^{\mathcal{P},\sigma} \tag{25}$$

holds. Here, by equations (20), (23)-(25), we have

$$s_{\tilde{r}}^{\mathcal{P}',\sigma} = f_{\tilde{r}}(\delta_{r_1}^{c,\mathrm{I}}(s_{r_1}^{\mathcal{P}',\sigma'}, m), \ldots, \delta_{r_l}^{c,\mathrm{I}}(s_{r_l}^{\mathcal{P}',\sigma'}, m)), \tag{26}$$

and also by equations (19), (21) and (22), we have

$$s_{\tilde{r}}^{\mathcal{P},\sigma} = \delta_{\tilde{r}}^{c,\mathrm{O}}(f_{\tilde{r}}(s_{r_1}^{\mathcal{P}',\sigma'}, \ldots, s_{r_l}^{\mathcal{P}',\sigma'}), m). \tag{27}$$

Therefore, it follows from equations (26), (27) and equation (4) of condition 2 that $s_{\tilde{r}}^{\mathcal{P},\sigma} = s_{\tilde{r}}^{\mathcal{P}',\sigma}$.

In case 1), both the classes $\Phi(\mathcal{P}_{\mathcal{R}}, \tilde{r})$ of $\mathcal{P}_{\mathcal{R}}$ and $\Phi(\mathcal{P}'_{\mathcal{R}}, \tilde{r})$ of $\mathcal{P}'_{\mathcal{R}}$ have state fields. As is the case with 2), by the induction hypothesis, equation (19) holds, and by the condition part of the lemma and lemma 4, equations (20) and (25) hold. With respect to the state of $\mathcal{P}_{\mathcal{R}}$ just after σ is input, equations (22) and (23) hold, and with respect to the state of $\mathcal{P}'_{\mathcal{R}}$ just after σ is input,

$$s_{\tilde{r}}^{\mathcal{P}',\sigma} = \delta_{\tilde{r}}^{c,\mathrm{O}}(s_{\tilde{r}}^{\mathcal{P}',\sigma'}, m'), \tag{28}$$

$$s_{r_j}^{\mathcal{P}',\sigma} = \delta_{r_j}^{c,\mathrm{I}}(s_{r_j}^{\mathcal{P}',\sigma'}, m') \tag{29}$$

hold for an appropriate message m' on channel c. Hence, by equations (20), (23), (25) and (29), $m = m'$ holds. Thus, it follows from equations (19), (22) and (28) that $s_{\tilde{r}}^{\mathcal{P},\sigma} = s_{\tilde{r}}^{\mathcal{P}',\sigma}$.

In case 3), with respect to the state of $\mathcal{P}_{\mathcal{R}}$ just after σ is input, by the generation of the PUSH-first prototype, both the response of the getter method of each class $\Phi(\mathcal{P}_{\mathcal{R}}, r_i)$ $(1 \leq i \leq l)$ and its internal state (if it is stored) become $s_{r_i}^{\mathcal{P},\sigma}$. Similarly, with respect to the state of $\mathcal{P}'_{\mathcal{R}}$ just after σ is input, by the generation of the PUSH-first prototype, both the response of the getter method of each class $\Phi(\mathcal{P}'_{\mathcal{R}}, r_i)$ $(1 \leq i \leq l)$ and its internal state (if it is stored) become $s_{r_i}^{\mathcal{P}',\sigma}$. Furthermore, by lemma 4 and the condition part of the lemma, we have

$$s_{r_i}^{\mathcal{P},\sigma} = s_{r_i}^{\mathcal{P}',\sigma}. \tag{30}$$

Therefore, by the generation of a PULL-containing prototype and equation (30), the responses of the getter methods of $\Phi(\mathcal{P}_{\mathcal{R}}, \tilde{r})$ and $\Phi(\mathcal{P}'_{\mathcal{R}}, \tilde{r})$ just after σ is input become

$$s_{\tilde{r}}^{\mathcal{P},\sigma} = f_{\tilde{r}}(s_{r_1}^{\mathcal{P},\sigma}, \ldots, s_{r_l}^{\mathcal{P},\sigma}) = f_{\tilde{r}}(s_{r_1}^{\mathcal{P}',\sigma}, \ldots, s_{r_l}^{\mathcal{P}',\sigma}) = s_{\tilde{r}}^{\mathcal{P}',\sigma}. \tag{31}$$

$\square$

Theorem 2. *Let $\mathcal{R} = \langle R, C, \rho, D, \tau, \mu, \Delta, s_0 \rangle$ be an arbitrary valid data transfer architecture model, and $\mathcal{P}_{\mathcal{R}}$ be any JAX-RS prototype generated from $\mathcal{R}$ and satisfying conditions 1 and 2. Then, $\mathcal{P}_{\mathcal{R}}^{\mathrm{PSH}}$ and $\mathcal{P}_{\mathcal{R}}$ satisfy*

$$\begin{aligned} \forall_{\sigma}.\forall_{r \in R}.\ &\{s_r \mid \exists_s.\mathcal{P}_{\mathcal{R}}^{\mathrm{PSH}}(s_0) \overset{\sigma}{\Rightarrow} \mathcal{P}_{\mathcal{R}}^{\mathrm{PSH}}(s) \overset{\mathrm{get}(r)/s_r}{\Longrightarrow} \mathcal{P}_{\mathcal{R}}^{\mathrm{PSH}}(s)\} \\ &= \{s'_r \mid \exists_{s'}.\mathcal{P}_{\mathcal{R}}(s_0) \overset{\sigma}{\Rightarrow} \mathcal{P}_{\mathcal{R}}(s') \overset{\mathrm{get}(r)/s'_r}{\Longrightarrow} \mathcal{P}_{\mathcal{R}}(s')\}. \end{aligned}$$

Proof. The theorem is proved by induction on the number $n = |R|$ of the resources and lemma 5.
(**basis**)
Assume that $\mathcal{R}$ has exactly one resource and exactly one channel as an I/O channel, that is, $C = \{c_{\mathrm{I/O}}\}$, $R = \{r\}$ and $\rho(c_{\mathrm{I/O}}, \mathrm{O}) = \{r\}$. Then, since there is no data transfer and always $\mathcal{P}_\mathcal{R} = \mathcal{P}^{\mathrm{PSH}}_\mathcal{R}$ holds, the lemma obviously holds.
(**induction step**)
If $n > 1$, then since the dataflow graph $G_\mathcal{R}$ of $\mathcal{R}$ has no strongly connected component, there exists at least one resource $\tilde{r}$ such that $\mathrm{Out}_\mathrm{C}(\tilde{r}) = \emptyset$. Let $\mathcal{R}'$ be the data transfer architecture model obtained by removing $\tilde{r}$ from $\mathcal{R}$. Then, since the number of resources of $\mathcal{R}'$ is $|R \setminus \{\tilde{r}\}| = n - 1$, by the induction hypothesis, the lemma holds for $\mathcal{R}'$. Furthermore, by lemma 5, the lemma also holds for $\mathcal{R}$. □

Theorem 3. *Let $\mathcal{R} = \langle R, C, \rho, D, \tau, \mu, \Delta, s_0 \rangle$ be an arbitrary valid data transfer architecture model, and $\mathcal{P}_\mathcal{R}$ be any JAX-RS prototype generated from $\mathcal{R}$ and satisfying conditions 1 and 2. For an arbitrary input sequence σ, $s_0 \underset{\mathcal{R}}{\overset{\sigma}{\Longrightarrow}} s$ holds if and only if there exists a state s' of $\mathcal{P}_\mathcal{R}$ such that $\mathcal{P}_\mathcal{R}(s_0) \overset{\widehat{\sigma}}{\Rightarrow} \mathcal{P}_\mathcal{R}(s') \overset{\mathrm{get}(r)/s(r)}{\Longrightarrow} \mathcal{P}_\mathcal{R}(s')$ holds for any resource $r \in R$.*

Proof. The theorem follows from the theorems 1 and 2. □